Optimizing a Coil Gun

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Abstract

In the building of coil guns, the standard basic approach is to simply guess the dimensions of the coil, hook up as many batteries as you can get your hands on, and fire it. However, this approach is inefficient and can be dangerous. A much better approach is that of quantitative analysis, in which users optimize the dimensions of the coil to the amount of power available. There are two approaches to this:

1. Running a simulation and educated guessing with an FEA system such as FemLab
   While this approach will theoretically work with every problem, it is much harder to use properly to get accurate results. Most people in our class used this approach, and the results varied widely.
2. Finding the analytic, closed form solution and using a findmax() function
   If all the math is done correctly, this method *will* give the correct answer and will usually be more efficient than (1). This is usually not done because the math is generally difficult and (1) requires no math at all. However, a CAS (computer algebra system) such as Mathematica or Maple so greatly simplifies the math that the problem becomes almost trivial.

We chose to take the second approach.

1 Introduction

We were given the task to design and build a 1 to 4 stage coil gun that fires a neodymium magnet as far as possible with a maximum power of 25Watts. A one-coil setup is shown in figure 1

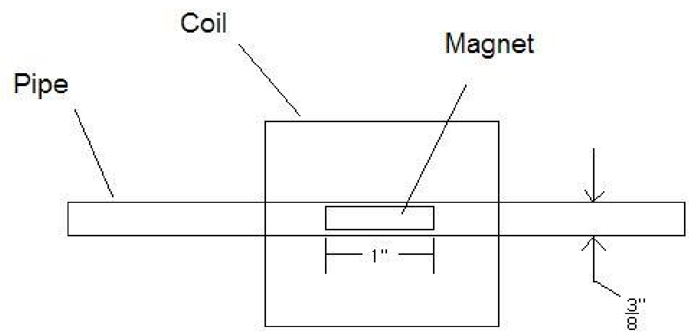

Figure 1: A one-coil apparatus for launching the magnet. When a current density passes through the coil, a magnetic field is created inside the coil. Since this field is slightly non-uniform, if the magnet is slightly offset from the center of the coil, it will feel a force and accelerate out of the coil.

To fire the farthest, the coil gun must impart the most kinetic energy to the magnet, and as we know:

$$KE = \int F\,dx \tag{1}$$

To find F on the magnet, we first examine F on a infinitesimally short magnet, then integrate over the length of the magnet, L_m, to find the total force. A small section of magnet can be taken to be equivalent to a small loop of wire with a current running through it. The axial magnetic field on the coil will exert a radial force, but the radial magnetic field exerts an axial force on the current loop, so:

$$F \propto B_{radial} \tag{2}$$

If we consider a small cylinder that is coaxial with the coil, we know that:

$$\int_0^R \int_0^{2\pi} B_{axial}(0)\, r\, dr d\theta - \int_0^{\Delta x} \int_0^{2\pi} B_{radial}(R)\, R\, dx d\theta - \int_0^R \int_0^{2\pi} B_{axial}(\Delta x)\, r\, dr d\theta = 0$$

$$B_{axial}(0) - B_{axial}(\Delta x) \propto \Delta x B_{radial}(R)$$

$$\frac{B_{axial}(0) - B_{axial}(\Delta x)}{\Delta x} \propto B_{radial}(R)$$

And letting $\Delta x \to 0$:

$$\frac{dB_{axial}}{dx} \propto B_{radial} \tag{3}$$

Combining 1, 2, and 3, we get a startling result:

$$KE \propto \int \frac{dB_{axial}}{dx}\,dx$$
$$KE \propto B_{axial} \tag{4}$$

So the total KE gained by a small slice of magnet as it exits the coil is simply proportional to the value of the axial magnetic field where it starts. Thus, to maximize the KE of the magnet, we need only maximize the B field in the coil[1]!

2 Optimizing the Coil

We must do several things to optimize the coil. First, we must optimize the dimensions of the coil to give us the highest magnetic field for 25 Watts. Second, we must pick the size of wire we wish to use.

2.1 Maximizing the Magnetic Field

To maximize the magnetic field from the coil over the magnet, we must first find the magnetic field as a function of x, the position in the coil; R_{out}, the outer diameter of the coil; and L_c, the length of the coil.

We begin by knowing that the B field a distance l along the axis from the center of a single coil of wire with radius r and current I is

$$B = \frac{\mu_0 I r^2}{2\left(l^2 + r^2\right)^{\frac{3}{2}}}$$

To find the B field from the whole coil, we let $I = J\,dA$ and integrate[2]. This gives us an expression for the magnetic field from the whole coil at any position along the axis. We can then integrate this over the magnet, which gives us a closed form solution to the total magnetic field produced by the coil summed over the the magnet as a function of only the length and radius of the coil. This is what we want to maximize.

When we plot this function over a reasonable range of R_{out} and L_c, we get the result shown in figure 2. As we can see, the function has a very clear global maximum and is in the correct order of magnitude for our coil. We can estimate the maximum by simply looking at the graph, but to get a high level of precision, we use our visual estimate as a starting point for Mathematica's built in maximization function. Doing this, we find the optimum dimensions of the coil to be;

$$R_{out} = 0.0193m = 1.93cm \tag{5}$$
$$L_c = 0.0333m = 3.33cm \tag{6}$$

[1] It's actually a little harder than this because we have a non-zero length magnet, but that simply means we need to maximize the integral of the B field over the whole magnet, which is trivial in a CAS.

[2] From this point on, the formulas are inconveniently complicated, and so we refer the reader to all six pages of the first attached Mathematica printout with the complete solution.

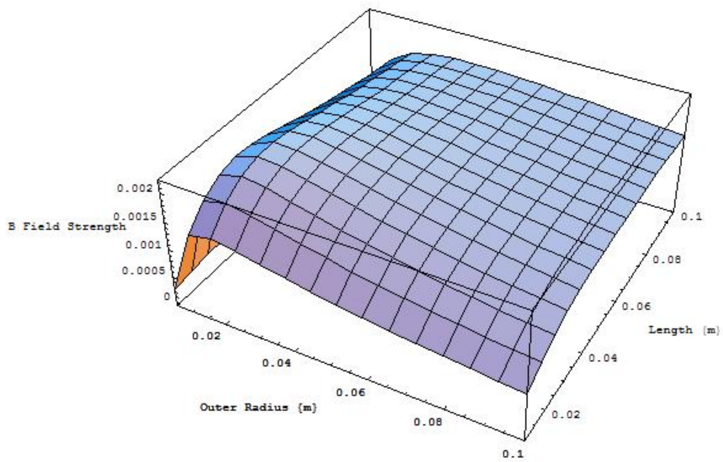

Figure 2: Total magnetic field acting on the magnet in the center of the coil plotted as a function of R_{out} and L_c.

2.2 Finding the Wire Gauge

To find the best wire diameter, we determine a formula for the total resistance of the wire used in the coil, set the resistance to some moderate value, solve for the diameter, and pick the gauge of wire closest to that diameter. For our implementation of this procedure, we refer the reader to all four pages of the second attached Mathematica printout. A range of wire gauges will work, so we are free to choose based solely on winding properties. We ended up choosing 22 gauge wire because it has a nice balance of flexibility and size. Using 22 gauge gives us a total resistance of 4 Ohms, a current draw of 2.5 Amps, and a necessary voltage of 10 Volts, giving us the required power of 25 Watts.

3 Building the Coil

To build the coil, we used custom-machined plastic end stops held in place with superglue to ensure that our coils were the correct length. We used a standard screw gun to wind the coils, and applied a thin coat of superglue every few layers to make sure the coil did not come apart while we were winding it. As we are busy people and did not have forever to make the coils exactly perfect, our wrapping job was less than ideal, though it was still much better than many groups' coils. One of the main problems we had with wrapping was that the wraps tended to traverse the coil to quickly, meaning that they wouldn't be perpendicular to the axis of the coil, which makes their contribution to the magnetic field weaker. We used two coils spaced approximately 1" apart with separate leads to give our gun a little extra oomph.

4 Results from a Single Coil

After we push our magnet into the coil to a point just past the center[3], our professors apply a voltage pulse of the value given above. This shoots the magnet out the end of the coil and into a marking bed from which we measure the total distance travelled. We use this distance to find the kinetic energy of the magnet and use that to find the efficiency of our coil gun.

4.1 Calculating Kinetic Energy

By measuring the marking bed, we find that:

$$d = 1.295m$$

The time taken for the magnet to fall to the marking bed is given by:

$$.6096m = \frac{1}{2}9.8t^2$$

$$t = .353s \tag{7}$$

So the muzzle velocity of the single coil is:

$$v = \frac{1.295m}{.353s} = 3.67m/s \tag{8}$$

By weighing the magnet, we find that its mass is .005kg, so the total kinetic energy is:

$$KE = \frac{1}{2}.005 \cdot 3.67^2 = 33.7mJ \tag{9}$$

4.2 Calculating Efficiency

We can find the efficiency of the coil by calculating the ratio of kinetic energy to total energy used by the coil. The total energy E is given by:

$$E = 25t \tag{10}$$

Where t is the time spent gaining the kinetic energy, or the time spent accelerating. To calculate this, we first find the distance at which the force on the magnet becomes negligible by slowly moving the magnet away from a powered coil until we could no longer feel the force. We measure this distance to be ≈ .075m. Assuming constant acceleration over this distance, we can use an equation of motion to find t:

$$d = \frac{1}{2}(v_{final} + v_{initial})t$$

[3]So that the magnet experiences the maximum magnetic field while still being forced to exit the correct direction.

$$.075 = \frac{1}{2} 3.67t$$
$$t = .04s \tag{11}$$

So, the total energy dissipated is

$$E = 25 \cdot .04 = 1J \tag{12}$$

Thus, the efficiency of a single coil apparatus is:

$$\frac{E_{kinetic}}{E_{total}} = \frac{.0337}{1} = 3.37\% \tag{13}$$

Though this seems remarkably low, we must remember that an extraordinary amount of energy is being lost to heating the coil. This is, in fact, about the efficiency we would expect from a single coil.

5 Results from Two Coils

The firing method for two coils is much the same as that for one coil; however, to ensure that we only use 25 Watts, we have to calculate when to switch power from the first coil to the second coil. Ideally, this is when the magnet has just crossed the center of the second coil. As it turns out, the distance between our coils is exactly the .075m measured above, so we have already found the time it takes to travel that distance. The professors provide us with an oscilloscope trace to observe how accurate our time calculation is, and our calculation of .04s is extremely close to the ideal.

5.1 Calculating Kinetic Energy

By measuring the marking bed, we find that the magnet travelled:

$$d = 1.7272m$$

Since the magnet falls the same distance as in equation 7:

$$v = \frac{1.7272m}{.353s} = 4.89m/s \tag{14}$$

Thus, the total kinetic energy of the magnet is:

$$KE = \frac{1}{2}.005 \cdot 4.89^2 = 59.7mJ \tag{15}$$

As we can see from this result and equation 9, when we use two coils, the total kinetic energy very nearly doubles. The 6mJ loss we see is probably due to either friction in the tube or more likely to small errors in the timing delay.

5.2 Calculating Efficiency

We can use equation **??** to find the total energy, but we must recalculate t because we are using a second coil. To do this, we can simply calculate t for the second coil, and add it to t for the first coil. Thus:

$$d = \frac{1}{2}(v_{final} + v_{initial})t$$

$$.075m = \frac{1}{2}(4.89m/s + 3.67m/s)t$$

$$t = .0175$$

So the total time the power is on is:

$$t_{total} = t_{first} + t_{second} = .04 + .018 = .058s \tag{16}$$

This means that the total energy dissipated is:

$$E = 25 \cdot .058 = 1.45J \tag{17}$$

Thus, the efficiency of the double coil apparatus is:

$$\frac{E_{kinetic}}{E_{total}} = \frac{.0597}{1.45} = 4.12\% \tag{18}$$

The efficiency goes up as we move from a single coil to a double coil because for each coil, the kinetic energy gained only depends on the distance travelled through the magnetic field while the coil is on. Total energy depends on *how much time* the field is on for. Thus, the amount of kinetic energy gained is constant regardless of the speed of the magnet, but the total energy is much lower when the magnet is moving faster because the magnet is in the field for less time. Thus, as we keep adding more and more coils, the apparatus will become more and more efficient because the magnet will be moving faster and faster through each successive coil.

6 Results and Conclusions

Our distance of 1.295m was significantly greater than any other coil in our section, leading us to believe that we were correct in our choice to use the analytic solution instead of using an FEA and educated guess-and-check method. Also, because we found the analytic solution, we would not change anything about the dimensions of our coil if we were going to revise it. The biggest problems with our coil are the wrapping density and the time delay for the two-coil apparatus. Increasing the wrapping density of the coils would give us closer to our ideal current density and would better align the wraps with the axis of the pipe, increasing the magnetic field. We are able to determine the time delay to within the tolerance of where we placed the magnet in the first coil, so we would need a more precise method for placing the magnet before we could refine the time delay much farther. Also, it would be fun to add a third and perhaps a fourth coil, to see how much kinetic energy we gain from each one. We already inadvertently calculated the time delay for a third coil spaced .075m away from the second coil in equation 15, and it would be easy to calculate the time delay for the fourth coil using a similar method.